Homework 1
Due: Friday, February 11

All homeworks are due at 11:59 PM on Gradescope.

Please do not include any identifying information about yourself in the handin, including your Banner ID.

Be sure to fully explain your reasoning and show all work for full credit.

Problem 1

At a local coffee shop, there are five coffee toppings: cold foam, caramel drizzle, whipped cream, cinnamon dolce sprinkles, and cocoa powder. The toppings are chosen respecting the following rules:

i. If cold foam isn’t chosen, caramel drizzle is chosen.

ii. If whipped cream and caramel drizzle both are chosen, then cocoa powder can’t be chosen.

iii. If cinnamon dolce sprinkles isn’t chosen then cocoa powder must be chosen.

Consider the following questions regarding choosing coffee toppings:

(a) Use one variable to represent choosing each coffee topping, and write logical expressions using $\land$, $\lor$, $\rightarrow$, and $\neg$ to demonstrate the above rules.

(b) If neither cinnamon dolce sprinkles nor cold foam is chosen, what is? What isn’t? Explain your logic.
(c) It’s Pete’s first day working at this local coffee shop! His first customer arrives and passes Pete a slip of paper with his coffee order. After venturing to the back of the shop, he discovers three different types of syrups: hazelnut, toffee nut, and vanilla.

Only one of the syrups will make the customer extremely happy if added, while the other two syrups will give the customer an allergic reaction. On the customer’s order, there is a cryptic message below each of the syrup options:

![CIRCLE YOUR SYRUP(S):]

HAZELNUT
CHOOSE THIS SYRUP TO MAKE MY DAY

TOFFEE NUT
DO NOT CHOOSE THIS SYRUP

VANILLA
DO NOT CHOOSE THE TOFFEE NUT SYRUP

Given that at least one of the messages is true, and at least one of them is false, which syrup should be used to ensure that Pete’s customer doesn’t get an allergic reaction?

**Problem 2**

For each of the following expressions, convert it to both CNF and DNF using the provided method (show all work!).

a. Expression: \((p \rightarrow q) \rightarrow r\)
   Method: using rewrite rules (hint: do DNF first)

b. Expression: \(p \rightarrow (q \rightarrow r)\)
   Method: using truth tables
Problem 3

(a) Although we have been using AND and OR gates with only two inputs, we can create AND and OR gates that take in more than two inputs. Under the hood, these can just be implemented with 2-input AND and OR gates.

Build a 3-input AND circuit using only 2-input AND gates, and build a 3-input OR circuit using only 2-input OR gates.

(b) Duncan and Bucky have spent many nights arguing about who makes better lattes. They decide to create a poll to determine what people think. However, to do so, they need a machine that can handle the poll data.

To help them out, construct a circuit using only AND, OR, and NOT gates. You may use gates of any number of inputs for this problem.

The circuit will take in three people’s answers (1 for Duncan and 0 for Bucky), and return the answer of the majority.

Please additionally explain how your circuit works.

(c) Joe and Pete are currently having a lot of trouble determining if the numbers 0, 1, 2, and 3 are odd or even. The solution? Build a circuit!

Model this problem as a circuit using only AND, OR, and NOT gates. You may use gates with more than one input. Create three input wires and one output wire. If an even number of inputs are on, the output should be off. If an odd number of input wires are on, the output should be on.

Please additionally explain how your circuit works.

Note: The output depends only on how many of the input wires are on. For example, if exactly one input is on, then the output should be on. It does not matter which input is on.

Food for thought\(^1\): What happens if you combined your circuit from (b) with your circuit from (c)? Think about the two individual outputs as a single, two piece output.

\(^1\)This is a Mind Bender for a tiny bit of extra credit.
Problem 4 (Mind Bender)

Mind Benders are extra credit problems intended to be more challenging than usual homework problems. They are denoted with a ≪ symbol. Occasionally, some parts from problem 1-3 might also be extra credit problems.

The $n$-queens problem involves a $n \times n$ chess board and $n$ queens. The goal is to place all of the queens on the board so that no queen is “threatening” another queen (able to attack it). Queens can move as far as they want vertically, horizontally, or diagonally.

(a) For $n = 1$ to 4, either construct a solution to the $n$-queens problem, or (briefly) justify why one does not exist.

(b) For an arbitrary positive integer $n$, describe how to construct a boolean formula which is satisfiable if and only if the $n$-queen problem is solvable. Your description should be explicit enough that anyone who has read the problem can follow them.

A hint is provided at the bottom of the page if you wish to utilize it.